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Abstract. On the basis of three-dimensional Monte Carlo simulations on the hadronic component of extensive air showers the electromagnetic component of extensive air showers has been investigated theoretically using approximative electromagnetic cascade theory. Results include the relation between mean total electron number and primary energy as well as mean lateral distribution of electrons. Fluctuations of parameters of the structure function, central density and width of density distribution in sets of showers with given initial conditions are presented and found to be strong.

The origin of observed deviations of electron density distribution from uniformity and axial symmetry is investigated with the help of calculated electron density matrices, the elements of which are electron numbers in squares of 10 cm × 10 cm. We find that contributions to such fluctuations due to the influence of primary mass number on the development of the hadronic component are small in comparison with contributions originating from fluctuations in the development of the electromagnetic cascades or from fluctuations connected with the detection of particles. Therefore, we conclude that, under conventional assumptions concerning nuclear interaction parameters, there is no possibility of inferring primary composition from observed multi-core structures in the electromagnetic component for vertical incidence of showers and for observations at sea level.

1. Empirical background

The core region of an extensive air shower is defined as the range, of distance r, a few metres from the shower axis. The distribution of electron density in this region, which will be the main object of the following considerations, is discussed in terms of (i) the structure function $n(r)$, which is the mean number of electrons per unit area as a function of distance from the shower axis and is usually characterized by parameters† such as central electron density Δ_e, 'steepness' $S(r) = -\mathrm{d}\,\lg\{n(r)\}/\mathrm{d}\,\lg r$ of the core at some 10 cm distance from the shower axis or width Γ_e of the core defined by $n(\Gamma_e) = \Delta_e/2{\cdot}718$, and (ii) deviations from the uniformity of distribution, which sometimes appear as 'subcores' or 'multicores'. The structure function has been investigated by Oda and Tanaka (1962) using a neon hodoscope covering an area of 12 m^2. These authors found that there is no unique lateral distribution for all showers of equal size and that about 5% of showers of size greater than 5×10^4 had steep cores with $S(r) > 1$ for $r < 10$ cm. These were interpreted as being composed of one flat distribution and single high-energy electromagnetic cascades originating from lower levels of the atmosphere. Improved studies of the structure function have been performed by Shibata *et al.* (1966) using a spark-chamber arrangement of sensitive area 20 m^2. Within a range of size between 3×10^4 and 10^6 they found that 5% of all single-core showers had steepness $S > 1$ in the region $0{\cdot}3 < r < 3$ m, while only 0·05% of all showers were steep down to a few centimetres distance from the shower axis.

By application of a neon hodoscope covering an area of 32 m^2 the steepness S of single-core showers was found to vary from 0·3 to 1·5 for distance $0{\cdot}5 < r < 4$ m from the axis

† Experimental results, to which we refer below, are generally expressed in terms of maximum electron density.

(Bagge *et al.* 1966). 10% of showers in the range of size between 10^4 and 10^5 were found to have maxima of electron density appearing like subcores above a uniform distribution (Trümper 1967). By requiring steepness $S > 1$ and more than 50 electrons in a circle of radius 10 cm this number of showers was reduced to 0·7% (J. Trümper and W. Büscher 1968, private communication). Subsequent closer investigations showed the majority of steep maxima to be due to nuclear-active bursts originating in the roof of their experimental arrangement (Böhm *et al.* 1968).

Comprehensive studies on electron concentration in air shower cores have also been performed with the help of an arrangement consisting of 64 scintillators, each 41 cm × 41 cm, covering a total area of 4 m × 4 m (Bray *et al.* 1964, Winn *et al.* 1965, Bray *et al.* 1966, Bakich *et al.* 1968, McCusker *et al.* 1969), confirming that there is no unique core structure for showers of equal size. It has been proposed by these authors that fluctuations in central electron density reflect primary chemical composition.

Deviations from the uniformity of electron density distribution, which are essentially stronger than Poissonian fluctuations and may therefore be referred to as subcores, were first studied by Heinemann and Hazen (1953) using an arrangement of ionization chambers. These authors found that some showers exhibit multi-core structure. Oda and Tanaka (1962) reported on three remarkable multi-core events. The subcores showed relative distances of the order of 1 m. They were interpreted as single prominent electromagnetic cascades with energies more than 10^{13} ev originating from an altitude not higher than about 4 km above sea level. Apparent transverse momenta were found to exceed 5 GeV/*c*. Similar results were obtained by Shibata *et al.* (1966), who found that about 2% of all showers of size of the order of 10^5 exhibit double-core structure in the sense that there is a subcore, the central density of which exceeds $\frac{1}{10}$ of the central density of the main core. Four examples of double-core showers have been analysed in detail with transverse momenta between 6·5 and 47 GeV/*c*. The largest observed distance between the two cores amounts to 3·20 m. Further measurements by these authors (Matano *et al.* 1968) resulted in a frequency of 3% multiple-core showers with transverse momenta above 5 GeV/*c*, the maximum being about 50 GeV/*c*. Similar conclusions were reached by the Osaka group (Miyake *et al.* 1968 a, b) using scintillators each of 0·25 m² in lattice arrangements consisting of 100 scintillators with unit distances of 5 m or 2·5 m for the investigation of regions distant from the core, and 48 scintillators closely packed for the inspection of the core region. Subcores are defined by $\Delta_2/\Delta_1 \geqslant 0{\cdot}33$. The rate of occurrence was about 1–2% for the 2·5 m separation unit and 0·1–0·2% for the 5 m unit, while an increase of this rate from 14 to 32% was found with the closely packed arrangement in the range of shower size from 10^4 to 10^6. The Tokyo and Osaka groups conclude that large values of transverse momentum are produced considerably more frequently than is expected under conventional assumptions.

Bray *et al.* (1964) reported on 44 single-core showers and 32 multi-core showers in the size range 10^5 to 10^6 and on two single-core showers and 21 multi-core showers for shower sizes greater than 10^6. According to these authors multiple and flat cores can be simulated by superposition of single-core showers and should not be explained as being due to single π^0 mesons. They argue that single-core showers are produced by proton primaries and multi-core showers by heavy primaries, and conclude that cosmic radiation at energies above 2×10^{15} ev seems to be richer in heavy primaries than at lower energies. Recent work of the Sydney group (Bakich *et al.* 1968, McCusker *et al.* 1969) leads to the consideration of an increase in mean transverse momentum with increasing collision energy as an explanation for the observed increase in the frequency of multi-core showers, similar to the proposals discussed above (Oda and Tanaka 1962, Shibata *et al.* 1966, Miyaka *et al.* 1968 a, b).

Multi-core showers have also been observed by Bagge *et al.* (1966) with the help of the aforementioned neon hodoscope. These authors registered several per cent of outstanding multi-core events, which were first thought to be due to heavy primaries (Trümper 1967), but later on the greater part was found to be caused by nuclear-active bursts in the roof structure of the laboratory (Böhm *et al.* 1968). Recently, Samorski *et al.* (1969) have compared observed deviations from a uniform structure function of electron numbers in squares of

$0{\cdot}21\ m^2$ in neon hodoscope pictures with simulated deviations randomly distributed according to Poisson's law. They conclude by application of such a scanning pattern that, within the limits of the statistics of 60 showers, the former deviations are quite consistent with the latter. According to these authors the results of McCusker *et al.* (1969) may be understood as non-essential fluctuations, in the sense defined below.

2. Monte Carlo procedures

In the present work we wish to distinguish between two contributions to the deviations from uniformity and axial symmetry of electron density distribution of individual air showers;

(i) Contributions which are inherent in the development of the hadronic cascade initiated by the primary particle and which in principle may contain information on primary mass number and/or on high-energy strong interaction properties ('essential fluctuations').

(ii) Contributions which are due to deviations from the mean development within individual electromagnetic cascades or which reflect Poissonian fluctuations in the response of the detector arrangements, and, furthermore, fluctuations of electron density which may be simulated by nuclear active particles from strong interactions within the detector arrangements or in the detector itself. The latter type of contributions to fluctuations of electron density neither gives information on primary composition nor on high-energy strong interaction properties ('non-essential fluctuations').

By means of the Monte Carlo method we wish to simulate type (i) fluctuations in extensive air shower experiments. Therefore we have performed Monte Carlo simulations implying stochastic phenomena, from which essential fluctuations originate, neglecting those phenomena which cause non-essential ones. This amounts to Monte Carlo simulations for the hadronic component, including a rather large number of interaction parameters, and to the mean development of electromagnetic cascades starting from the decay of π° mesons. Tables containing results of approximative cascade theory (Kamata and Nishimara 1958)—longitudinal development of total electron number (approximation B) and lateral distribution of electron density (core approximation using Molière's results)—have been recalculated by Thielheim and Siewers (1969) applying initial conditions corresponding to the decay of π° mesons of given energy.

If essential fluctuations of electron density are found to be smaller or at least not greater than non-essential fluctuations—and this will in fact be shown in § 3—it turns out to be meaningful to consider the structure functions $n(r)$ within the core region of individual showers. The mean structure function $\langle n(r)\rangle_{E_0,A}$ will then be calculated for sets of showers with given initial conditions E_0 and A. Parameters Δ_e and Γ_e characterizing the mean structure function $\langle n(r)\rangle_{E_0,A}$ will be obtained as functions of E_0 and A; fluctuations of these parameters will be investigated.

The model of high-energy interactions used in the Monte Carlo calculations presented here has been described in detail in a previous paper (Thielheim and Beiersdorf 1969) and need not be reproduced here.† A-induced showers of primary energy E_0 are simulated by superimposing at random A p-induced showers of primary energy E_0/A. Some of our previous results have shown that this procedure leads to a resonable compromise between various fragmentation models, which may be applied to the first interaction (Thielheim *et al.* 1968 a).

† The model includes excitation of the surviving baryon in nucleon–nucleus collisions (excited baryon state Δ (1236), excitation probability 0·75) and the Cocconi–Koester–Perkins type of pion production (mean elasticity 0·67 with respect to the surviving baryon state, corresponding to 0·56 with respect to the surviving nucleon ground state for nucleon–nucleus collisions and 0·20 for pion–nucleus collisions; mean transverse momenta 0·55 GeV/c for the excited baryon state, 0·42 GeV/c for the nucleon in case of no excitation and 0·33 GeV/c for pions; mean free paths 80 g cm^{-2} for nucleons and 100 g cm^{-2} for pions).

3. Results and conclusions

The relation between mean total electron number $\langle N_e \rangle$, primary energy E_0 and primary atomic number A, which is rather fundamental in air shower physics since it is used for the estimation of primary energy from observed shower size, is found to have the form of a power law

$$\langle N_e \rangle_{E_0,A} = 0{\cdot}0157 E_0^{1{\cdot}18} A^{-0{\cdot}18} \qquad (E_0 \text{ in GeV}) \tag{1}$$

and is represented in figure 1. The fact that $\langle N_e \rangle_{E_0,A}$ increases stronger than proportionally with increasing E_0 is due to the decrease in mean height of the shower maximum above

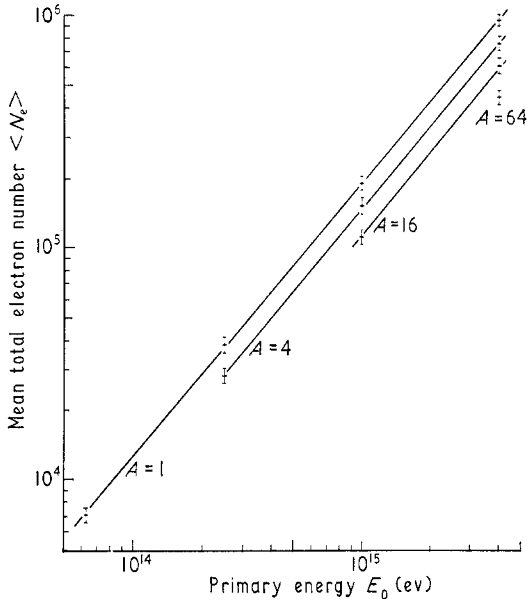

Figure 1. Mean total electron number $\langle N_e \rangle$ as a function of primary energy E_0 for various primary mass numbers $A = 1, 4, 16$ and 64.

sea level with increasing primary energy in the range of shower size considered here. Since A-induced showers are simulated by the superposition of p-induced showers of energy E_0/A, the above relationship has the transformation property

$$\langle N_e \rangle_{E_0,A} = A \langle N_e \rangle_{E_0/A,1}. \tag{2}$$

Absolute values of $\langle N_e \rangle_{E_0,A}$ for given E_0 and A are, of course, rather strongly model dependent. Correspondingly, results of de Beer *et al.* (1966), Murthy (1967), McCusker *et al.* (1968, 1969) and Ogita *et al.* (1968) differ from (1), but only by a factor of 2 or less owing to somewhat different assumptions on high-energy strong interaction properties. Results of Bradt and Rappaport (1968) exhibit somewhat stronger deviations from (1).

The exponent of primary energy E_0 in relation (1), which is found to be 1·18, agrees quite well with results of de Beer *et al.* (1966), who obtained about 1·13, and of Murthy (1967), who obtained about 1·16.

Standard deviations σ of $\lg N_e$ are 0·418 for $E_0 = 6{\cdot}25 \times 10^{13}$ ev, 0·292 for $E_0 = 2{\cdot}5 \times 10^{14}$ ev, 0·264 for $E_0 = 10^{15}$ ev and 0·230 for $E_0 = 4 \times 10^{15}$ ev. Decrease of standard deviation with increasing primary energy reflects the approach of the shower maximum to the level of observation. There is a general agreement of our data, and especially of the tendency mentioned above, with results of de Beer *et al.* (1966), Bradt and Rappaport (1968) and Ogita *et al.* (1968).

The mean lateral distribution of electron density $\langle n(r)\rangle_{E_0,A}m^{-2}$ has been calculated for sets of about 100 p-induced showers each at primary energies $E_0 = 6{\cdot}25\times10^{13}$ ev, $2{\cdot}5\times10^{14}$ ev, 10^{15} ev, and 4×10^{15} ev. Mean structure functions of A-induced showers, presented in figure 2, are obtained from those of p-induced showers using the relation

$$\langle n(r)\rangle_{E_0,A} = A\langle n(r)\rangle_{E_0/A,1}, \tag{3}$$

in agreement with the procedure of superposition. As may be verified from figure 2, the mean structure function steepens with decreasing primary mass number at fixed primary energy, or with increasing primary energy at fixed primary mass number. This tendency may be discussed in terms of the mean central electron density relative to the mean total electron number $\langle\Delta_e\rangle_{E_0,A}/\langle N_e\rangle_{E_0,A}$ presented in table 1. The reference area for the calculation of the central electron density Δ_e is 20 cm × 20 cm. According to present data

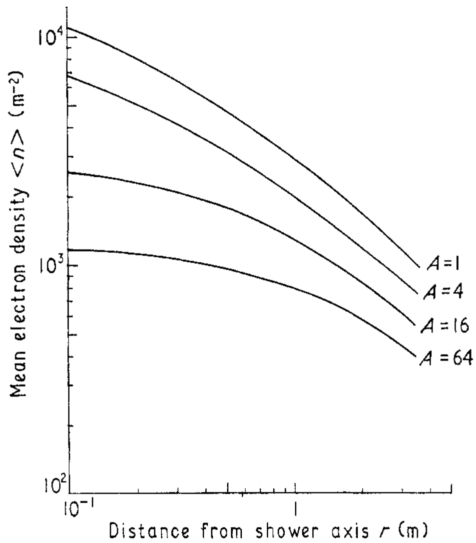

Figure 2. Mean lateral distribution of electrons $\langle n(r)\rangle$ in the core of air showers of fixed primary energy $E_0 = 4\times10^{15}$ ev and various primary mass numbers $A = 1, 4, 16$ and 64.

shown in the first row of table 1, $\langle\Delta_e\rangle_{E_0,A}/\langle N_e\rangle_{E_0,A}$ increases approximately proportionally to $(E_0/A)^{1/2}$ for $E_0/A \lesssim 10^{15}$ ev, with an indication of saturation for higher values of E_0/A. This clearly does not agree—at least in this region of E_0/A—with $\langle\Delta_e/N_e\rangle_{E_0,A} \propto 1/A$ as formerly proposed by Bray *et al.* (1964) and Winn *et al.* (1965). Our conclusion is qualitatively consistent with the Monte Carlo results of McCusker *et al.* (1968, 1969), which are presented in the second row of table 1 for comparison.†

Δ_e/N_e and N_e of individual showers of primary energy $E_0 = 4\times10^{15}$ ev and various primary mass numbers, are shown in figure 3. In this set of Monte Carlo showers, there are a hundred p-induced showers (●), twenty-five α-induced showers (+), six ($A = 16$)-induced showers (Δ) and two ($A = 64$)-induced showers (□). As may be verified from the histograms on the left of figure 3, showing projected Δ_e/N_e distributions for p-induced showers and for α-induced showers separately, the range of Δ_e/N_e values extends from

† The results of McCusker *et al.* concern maximum electron density. This difference in the definition of Δ_e does not alter the argument given above.

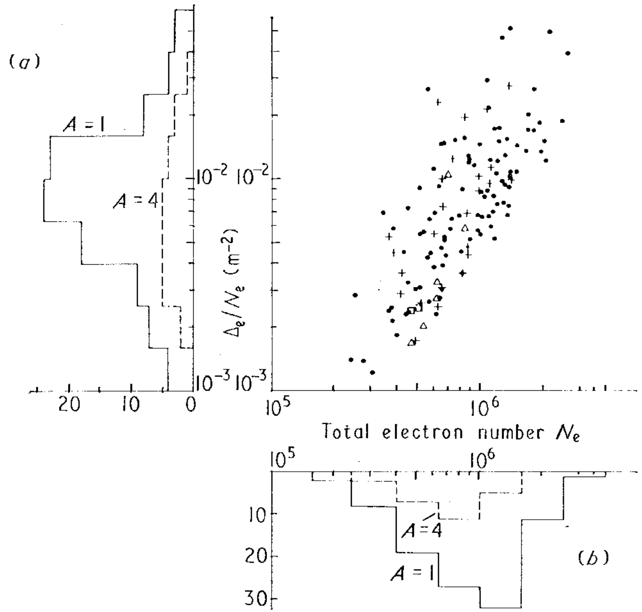

Figure 3. Values of the quotient Δ_e/N_e against values of N_e for individual showers of fixed primary energy $E_0 = 4 \times 10^{15}$ ev and various primary mass numbers $A = 1$, 4, 16 and 64. Total distributions of Δ_e/N_e and N_e are given in histograms on the left and below for p- and α-induced showers, separately. ● $A = 1$, 100 events; + $A = 4$, 25 events; △ $A = 16$, 6 events; □ $A = 64$, 2 events.

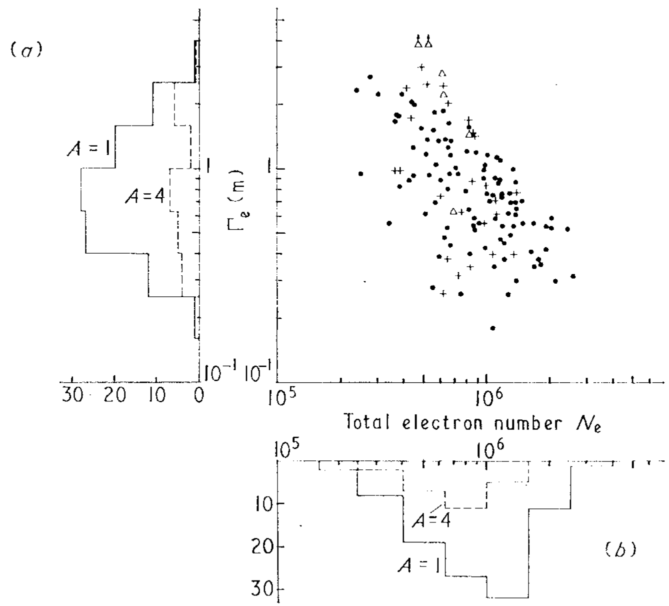

Figure 4. Values of distance Γ_e from the shower axis, where the electron density has decayed to Δ_e/e, against values of N_e for individual showers of fixed primary energy $E_0 = 4 \times 10^{15}$ ev and various primary mass numbers $A = 1$, 4 and 16. Total distributions of Γ_e and N_e are given in the histograms on the left and below for p- and α-induced showers, separately. ● $A = 1$, 100 events; + $A = 4$, 25 events; △ $A = 16$, 6 events.

Table 1. Values of $\langle\Delta_e\rangle/\langle N_e\rangle$ for various values of primary energy per nuclear E_0/A

E_0/A (ev)	$1{\cdot}8^{13}$†	$6{\cdot}25^{13}$	$2{\cdot}5^{14}$	1^{15}	4^{15}	Area of reference	
$\langle\Delta_e\rangle/\langle N_e\rangle$ (m^{-2})	—	$2{\cdot}36^{-3}$	$4{\cdot}53^{-3}$	$1{\cdot}02^{-2}$	$1{\cdot}36^{-2}$	0·2 m × 0·2 m	present data
$\langle\Delta_e\rangle/\langle N_e\rangle$ (m^{-2})	$2{\cdot}52^{-3}$	4^{-3}	$7{\cdot}56^{-3}$	$7{\cdot}76^{-3}$	—	0·5 m × 0·5 m	McCusker *et al.* (1968)

Δ_e is obtained by averaging over the central area of 20 cm × 20 cm (second row) while the values of the Sydney group (third row) refer to the area of 50 cm × 50 cm with maximum value of electron density.

† The superscript denotes the power of 10 by which the number is to be multiplied.

$1{\cdot}22\times10^{-3}$ m^{-2} to $5{\cdot}14\times10^{-2}$ m^{-2} within this set. Obviously, the dependence of $\langle\Delta_e\rangle_{E_0,A}/\langle N_e\rangle_{E_0,A}$ on A is strongly masked by fluctuations. Therefore, $\langle\Delta_e\rangle_{E_0,A}/\langle N_e\rangle_{E_0,A}$ may not be considered to be a good parameter for the determination of primary chemical composition, as was formerly discussed by Bray *et al.* (1964) and Winn *et al.* (1965).

Similar conclusions are reached concerning the central width Γ_e of the structure function $n(r)$, which, for the same set of simulated showers, is presented in figure 4.

Parameters Δ_e and Γ_e of the structure function $n(r)$ exhibit extremely strong fluctuations in sets of showers with given initial conditions. These fluctuations tend to diminish the information on primary mass number contained in the measurement of such parameters. In other words, the observed fluctuations of these parameters in sets of showers of given size are due rather to fluctuations in the development of extensive air showers than to contributions of different primary mass numbers (at least as long as conservative assumptions on primary mass composition are made). This result may be understood as a consequence of the fact that the main contribution to the core region of the electromagnetic component at sea level originates from π° mesons within a layer of atmospheric height at a relatively small altitude‡, such that almost all fluctuations within the hadronic component are reflected in fluctuations of parameters Δ_e and Γ_e discussed above. At this level of the atmosphere lateral dissipation of high-energy hadrons due to the preceding diffusion of leading baryons is not so strong as to reflect influence of heavy primaries in the distribution of these parameters under conservative assumptions on transverse momenta.

In this context we should mention earlier results of longitudinal Monte Carlo simulations (Thielheim *et al.* 1968 b), which were performed using various models of high-energy strong interactions. This work resulted in values P for the probability of p-induced showers to have at least one prominent electromagnetic cascade, which by superposition on the general distribution of electrons in the air shower would give rise to a 'steep core'. Under conventional assumptions P was found to be of the order of 1%, which is quite consistent with present results under given statistics.

Deviations of electron distribution from uniformity and axial symmetry ('essential fluctuations') within individual Monte Carlo showers are investigated by inspecting electron density matrices, the elements of which are electron numbers in squares of 10 cm × 10 cm. Each matrix contains 50 × 50 elements corresponding to a total area of 5m × 5 m. The centre of the matrix is defined by extrapolation of the primary particle trajectory. Such matrices have been prepared for a set of 133 showers of primary energy 4×10^{15} ev comprising a hundred p-induced showers, twenty-five α-induced showers, six (A = 16)-induced showers and two (A = 64)-induced showers.

As an example a small central fragment of one matrix is shown in figure 5. This is one of the p-induced showers. Although this Monte Carlo shower is the one exhibiting the strongest deviations from regular distribution within the set mentioned above, it is quite obvious that essential fluctuations are very small and would certainly not prevail over additional non-essential fluctuations. Thus, the double-core structure, for which there is an indication in this example, would clearly be a non-observable one. We may therefore

‡ The existence of such an atmospheric layer was demonstrated by previous results of Thielheim and Karius (1966) as far as high energy π° mesons were concerned.

conclude that under conventional assumptions on high-energy strong interaction properties, especially transverse momentum distribution, essential fluctuations are small in comparison with non-essential ones. Thus, there seem to be two alternative possibilities for the explanation of fluctuation phenomena, which have been observed by various authors and which were referred to as multi-core events in § 1. One possibility is based on extremely high mean transverse momenta in strong interactions, the other is the interpretation as non-essential fluctuations. This latter point of view is illustrated by recent results of the Kiel air shower experiment, which were discussed previously. Within the limits of given statistics, results of present Monte Carlo simulations concerning the smallness of essential fluctuations are quite in agreement with earlier works by Thielheim *et al.* (1968 c), in which the probability P for A-induced showers of given primary energy E_0 having at least two prominent electromagnetic cascades of given relative transverse momentum was calculated and found to be typically well below 1%.

Monte Carlo calculations performed by McCusker *et al.* (1968, 1969) resulted in considerable percentages of multi-core structures induced by heavy primaries. These authors propose that agreement with experiment is reached under the assumption that the chemical composition of primary radiation in the range of shower size of 10^4 and 10^5 is similar to those at low energies. Ogita *et al.* (1968) have simulated p-induced showers applying rather high values of mean transverse momentum. Their calculations also resulted in considerable percentages of multi-core showers. Neverthess no explanation is given for rather outstanding events on the basis of their assumptions.

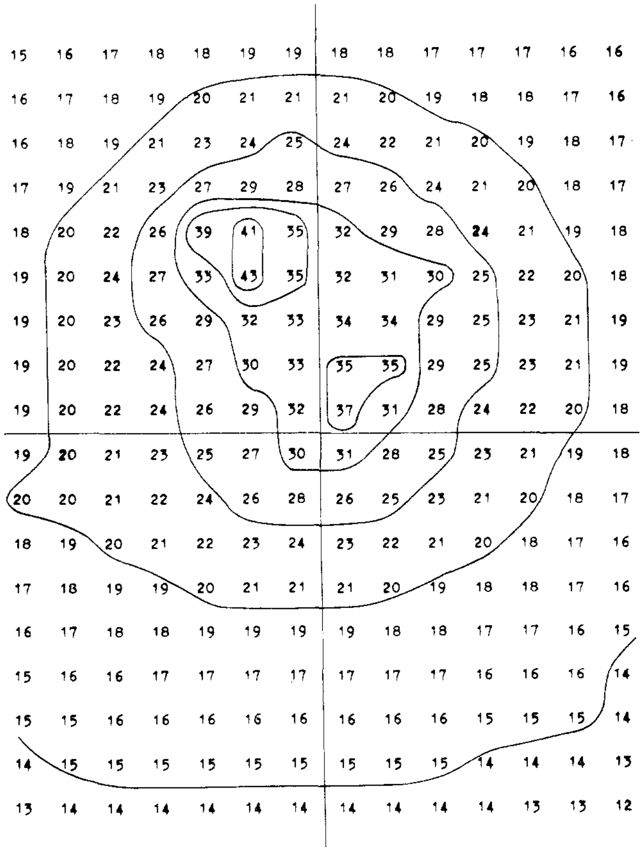

Figure 5. Central part of electron density matrix of a p-induced shower of primary energy $E_0 = 4 \times 10^{15}$ ev showing the greatest irregularities found in the simulated collection of 133 showers of this primary energy ($N_e = 6{\cdot}57 \times 10^5$).

119	126	133	140	145	147	149	151	155	146	139	132	126	120
128	138	144	154	161	163	166	168	168	163	154	146	137	130
138	148	158	168	178	186	192	194	188	182	171	163	151	142
146	160	174	188	202	218	222	222	218	210	193	179	166	158
156	174	195	214	237	256	267	267	262	245	220	199	180	167
167	190	217	243	277	320	337	335	315	292	255	223	196	175
177	203	238	273	332	406	453	450	394	345	306	251	211	186
184	221	254	303	387	495	645	633	504	400	330	264	221	192
188	221	259	319	419	582	1039	1039	635	446	336	276	224	195
185	212	252	314	409	553	1043	1093	661	458	333	267	223	192
179	205	241	293	366	486	582	621	514	398	306	257	219	187
170	193	224	267	312	370	432	441	400	330	274	238	206	182
161	182	207	238	264	294	319	325	302	278	242	214	190	172
151	167	185	206	227	241	256	262	247	232	213	193	175	158
141	154	172	184	199	208	215	216	209	200	186	173	163	146
131	142	154	164	172	181	190	188	180	172	164	156	148	135
122	132	139	145	153	158	163	161	157	152	147	143	135	125
116	122	127	132	137	140	143	142	140	137	134	129	122	116

Figure 6. Central part of electron density matrix of a p-induced shower of primary energy $E_0 = 4 \times 10^{15}$ ev showing the highest value of Δ_e ($N_e = 2{\cdot}13 \times 10^6$).

2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	3	3	3	3	3	3	3	3	2	2	2
2	2	2	3	4	3	3	3	3	3	3	2	2	2
2	2	3	3	3	3	3	3	3	3	3	2	2	2
2	2	2	3	3	3	3	3	3	3	3	2	2	2
2	2	2	3	3	3	3	3	3	3	3	2	2	2
2	2	3	3	3	3	3	3	3	3	3	2	2	2
2	2	3	3	3	3	3	3	3	3	3	2	2	2
2	2	3	3	3	3	3	3	3	3	3	2	2	2
2	2	3	3	3	3	3	3	3	3	2	2	2	2
2	2	2	3	3	3	3	3	3	3	2	2	2	2
2	2	2	3	3	3	3	3	3	2	2	2	2	2
2	2	2	2	3	3	3	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2

Figure 7. Central part of electron density matrix of a p-induced shower of primary energy $E_0 = 4 \times 10^{15}$ ev showing the lowest value of Δ_e ($N_e = 2{\cdot}40 \times 10^5$).

For comparison with our calculations one has to consider that the aforementioned calculations of McCusker *et al.* (1968, 1969) and Ogita *et al.* (1968) are concerned with showers of smaller central electron density due to lower primary energy and that criteria referring to relative particle density have been applied. Multi-core structures, which have been found under such assumptions, may generally not be strong enough to be discriminated significantly from Poissonian fluctuations.

The existence of well-defined structure functions, which exhibit axial symmetry, should be understood as being due to the superposition of a large number of electromagnetic cascades in individual showers rather than to the predominance of one electromagnetic cascade. For example, at a primary energy of 10^{15} ev typically 220 cascades contribute more than ten electrons to the matrix covering 5 m × 5 m.

Two more examples of electron density matrices are presented in figures 6 and 7, which demonstrate the existence of a well-defined structure function within individual showers, leaving aside non-essential fluctuations, as well as the existence of a wide range of variation of parameters such as Δ_e and Γ_e within a set of showers with given initial conditions.

Acknowledgments

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